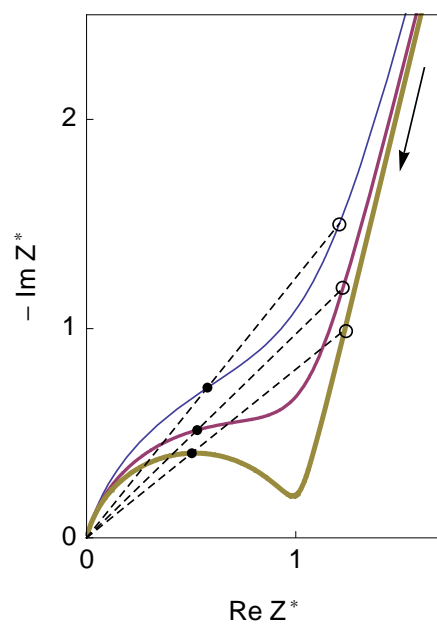


# Handbook of Electrochemical Impedance Spectroscopy



## ELECTRICAL CIRCUITS CONTAINING CPEs

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September 6, 2007



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# Chapter 1

## Circuits containing one CPE

### 1.1 Constant Phase Element (CPE), symbol Q

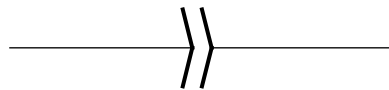


Figure 1.1: Most often used symbol for CPE (see also the Appendix A).

$$Z = \frac{1}{Q (i\omega)^\alpha}, \quad \text{Re } Z = \frac{c_\alpha}{Q \omega^\alpha}, \quad \text{Im } Z = -\frac{s_\alpha}{Q \omega^\alpha}$$
$$c_\alpha = \cos\left(\frac{\pi\alpha}{2}\right), \quad s_\alpha = \sin\left(\frac{\pi\alpha}{2}\right)$$
$$|Z| = \frac{1}{Q \omega^\alpha}, \quad \phi_Z = -\frac{\pi\alpha}{2}$$

The Q unit ( $\text{F cm}^{-2} \text{s}^{\alpha-1}$ ) depends on  $\alpha$ .

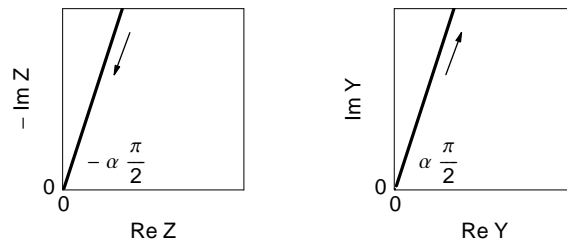


Figure 1.2: Nyquist diagram of the impedance and admittance for the CPE element, plotted for  $\alpha = 0.8$ . The arrows always indicate the increasing frequency direction.

## 1.2 Circuit (R+Q)

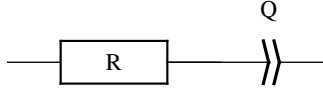


Figure 1.3: Circuit (R+Q).

### 1.2.1 Impedance

$$Z(\omega) = R + \frac{1}{Q(i\omega)^\alpha}, \quad \text{Re } Z = R + \frac{c_\alpha}{Q\omega^\alpha}, \quad \text{Im } Z = -\frac{s_\alpha}{Q\omega^\alpha}$$

### 1.2.2 Reduced impedance

$$Z^*(\omega) = \frac{Z(\omega)}{R} = 1 + \frac{1}{\tau(i\omega)^\alpha}, \quad \tau = RQ$$

The  $\tau$  unit depends on  $\alpha$ :  $u_\tau = s^\alpha$ .

$$Z^*(u) = 1 + \frac{1}{(iu)^\alpha}, \quad u = \omega\tau^{1/\alpha}$$

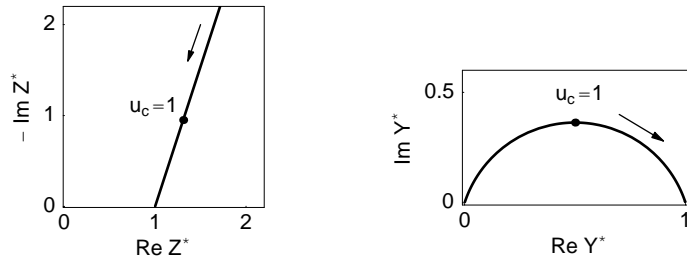


Figure 1.4: Nyquist diagram of the reduced impedance and admittance ( $Y^* = RY$ ) for the (R+Q) circuit, plotted for  $\alpha = 0.8$ .

### 1.3 Circuit (R/Q)

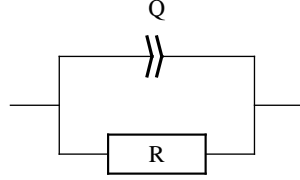


Figure 1.5: Circuit (R/Q).

#### 1.3.1 Impedance

$$Z(\omega) = \frac{R}{1 + \tau (i\omega)^\alpha}; \quad \tau = RQ$$

$$\operatorname{Re} Z(\omega) = \frac{R(1 + \tau \omega^\alpha c_\alpha)}{1 + \tau^2 \omega^{2\alpha} + 2\tau \omega^\alpha c_\alpha}; \quad \operatorname{Im} Z(\omega) = -\frac{R\tau \omega^\alpha s_\alpha}{1 + \tau^2 \omega^{2\alpha} + 2\tau \omega^\alpha c_\alpha}$$

#### 1.3.2 Reduced impedance

$$Z^*(\omega) = \frac{Z(\omega)}{R} = \frac{1}{1 + \tau (i\omega)^\alpha}; \quad \tau = RQ$$

$$\operatorname{Re} Z^*(\omega) = \frac{1 + \tau \omega^\alpha c_\alpha}{1 + \tau^2 \omega^{2\alpha} + 2\tau \omega^\alpha c_\alpha}; \quad \operatorname{Im} Z^*(\omega) = -\frac{\tau \omega^\alpha s_\alpha}{1 + \tau^2 \omega^{2\alpha} + 2\tau \omega^\alpha c_\alpha}$$

$$\frac{d\operatorname{Im} Z^*(\omega)}{d\omega} = \frac{\alpha \tau \omega^{-1+\alpha} (-1 + \tau^2 \omega^{2\alpha}) s_\alpha}{(1 + \tau^2 \omega^{2\alpha} + 2\tau \omega^\alpha c_\alpha)^2} = 0 \Rightarrow \omega_c^\alpha = 1/\tau \quad [3]$$

$$\operatorname{Re} Z^*(\omega_c) = 1/2, \quad \operatorname{Im} Z^*(\omega_c) = -\frac{s_\alpha}{2(1 + c_\alpha)}$$

$$\alpha = \frac{2}{\pi} \arccos \left( -1 + \frac{2}{1 + 4 \operatorname{Im} Z^*(\omega_c)^2} \right)$$

$$Z^*(u) = \frac{1}{1 + (iu)^\alpha}, \quad u = \omega \tau^{1/\alpha}$$

(Fig. 1.6)

#### 1.3.3 Pseudocapacitance

The value of the pseudocapacitance  $C$  ( $C/F \text{ cm}^{-2}$ ) for the (R/C) circuit giving the same characteristic frequency than that of the (R/Q) circuit (Fig. 1.7) is obtained from:

$$\omega_c = \frac{1}{(RQ)^{1/\alpha}} = \frac{1}{RC} \Rightarrow C = Q^{\frac{1}{\alpha}} R^{\frac{1}{\alpha}-1}$$

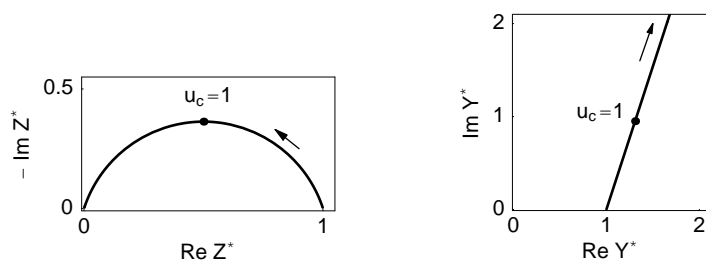


Figure 1.6: Nyquist diagram of the reduced impedance (depressed semi-circle [12]) and admittance ( $Y^* = RY$ ) for the (R/Q) circuit, plotted for  $\alpha = 0.8$ .

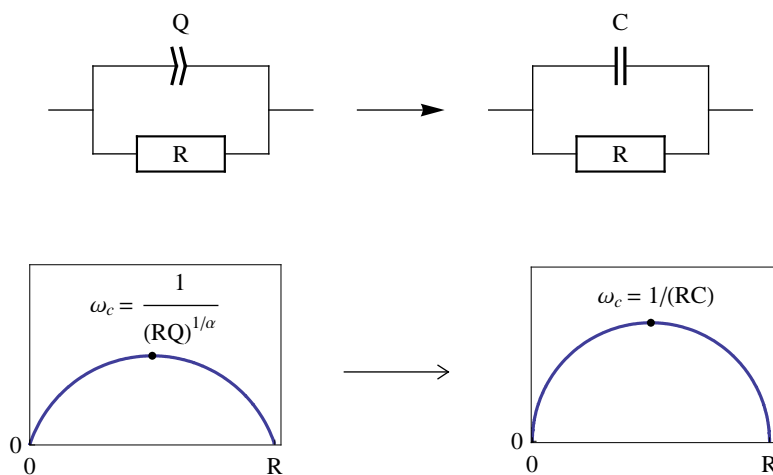


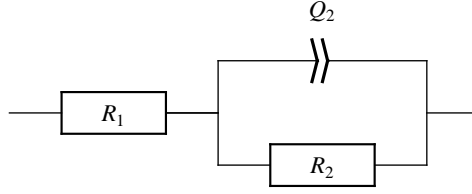
Figure 1.7: (R/Q) and (R/C) circuits with the same characteristic frequency at the apex (or summit) of impedance arc.

#### 1.4 Circuit (R/Q)+(R/Q)+ .. (Voigt)

$$Z(\omega) = \sum_{i=1}^{n_{RQ}} \frac{R_i}{1 + \tau_i (i\omega)^{\alpha_i}}; \tau_i = R_i Q_i$$

$$\text{Re } Z(\omega) = \sum_{i=1}^{n_{RQ}} \frac{R_i (1 + \tau_i \omega^{\alpha_i} c_{\alpha i})}{1 + \tau_i^2 \omega^{2\alpha_i} + 2\tau_i \omega^{\alpha_i} c_{\alpha i}}$$

$$\text{Im } Z(\omega) = - \sum_{i=1}^{n_{RQ}} \frac{R_i \tau_i \omega^{\alpha_i} s_{\alpha i}}{1 + \tau_i^2 \omega^{2\alpha_i} + 2\tau_i \omega^{\alpha_i} c_{\alpha i}}$$

Figure 1.8: Circuit ( $R_1 + (R_2/Q_2)$ ).

## 1.5 Circuit ( $R_1 + (R_2/Q_2)$ )

### 1.5.1 Impedance

$$Z(\omega) = R_1 + \frac{1}{(i\omega)^{\alpha_2} Q_2 + \frac{1}{R_2}}$$

$$Z(\omega) = \frac{(R_1 + R_2) (1 + (i\omega)^{\alpha_2} \tau_2)}{1 + (i\omega)^{\alpha_2} \tau_1}, \quad \tau_1 = R_2 Q_2, \quad \tau_2 = \frac{R_1 R_2 Q_2}{R_1 + R_2}$$

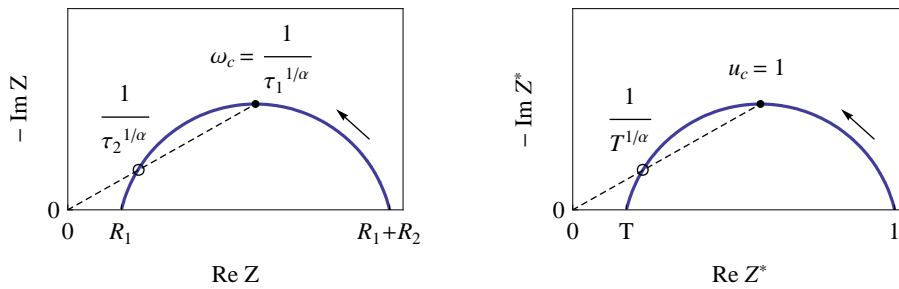
### 1.5.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1 + R_2} = \frac{1 + T (i u)^{\alpha_2}}{1 + (i u)^{\alpha_2}} \quad (1.1)$$

$$u = \tau_1^{1/\alpha_2} \omega, \quad T = \tau_2/\tau_1 = R_1/(R_1 + R_2) < 1$$

$$\operatorname{Re} Z^*(u) = \frac{T c_\alpha u^{\alpha_2} + c_\alpha u^{\alpha_2} + T u^{2\alpha_2} + 1}{2 c_\alpha u^{\alpha_2} + u^{2\alpha_2} + 1}$$

$$\operatorname{Im} Z^*(u) = -\frac{(1 - T) u^{\alpha_2} s_\alpha}{2 c_\alpha u^{\alpha_2} + u^{2\alpha_2} + 1}$$

Figure 1.9: Nyquist diagrams of the impedance and reduced impedance for the ( $R_1 + (R_2/Q_2)$ ) circuit.

## 1.6 Circuit ( $\mathbf{R}_1/(\mathbf{R}_2+\mathbf{Q}_2)$ )

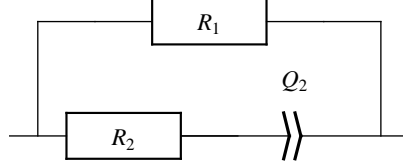


Figure 1.10: Circuit ( $\mathbf{R}_1/(\mathbf{R}_2+\mathbf{Q}_2)$ ).

### 1.6.1 Impedance

$$Z(\omega) = \frac{R_1 (1 + \tau_2 (i\omega)^{\alpha_2})}{1 + \tau_1 (i\omega)^{\alpha_2}}, \quad \tau_1 = (R_1 + R_2) Q_2, \quad \tau_2 = R_2 Q_2$$

$$\operatorname{Re} Z(\omega) = \frac{R_1 \left( \cos\left(\frac{\pi\alpha_2}{2}\right) (\tau_1 + \tau_2) \omega^{\alpha_2} + \tau_1 \tau_2 \omega^{2\alpha_2} + 1 \right)}{\tau_1 \left( \tau_1 \omega^{\alpha_2} + 2 \cos\left(\frac{\pi\alpha_2}{2}\right) \right) \omega^{\alpha_2} + 1}$$

$$\operatorname{Im} Z(\omega) = -\frac{\omega^{\alpha_2} \sin\left(\frac{\pi\alpha_2}{2}\right) R_1 (\tau_1 - \tau_2)}{\tau_1 \left( \tau_1 \omega^{\alpha_2} + 2 \cos\left(\frac{\pi\alpha_2}{2}\right) \right) \omega^{\alpha_2} + 1}$$

### 1.6.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{1 + T (iu)^{\alpha_2}}{1 + (iu)^{\alpha_2}}$$

$$u = \tau_1^{1/\alpha_2} \omega, \quad T = \tau_2/\tau_1 = R_2/(R_1 + R_2) < 1$$

cf. Eq. (1.1) and Fig. 1.9.

## 1.7 Transformation formulae between( $\mathbf{R}+(\mathbf{R}/\mathbf{Q})$ ) and ( $\mathbf{R}/(\mathbf{R}+\mathbf{Q})$ )

### 1.7.1 $\alpha_{21} = \alpha_{22}$

Transformations formulae ( $\mathbf{R}+(\mathbf{R}/\mathbf{Q}) \rightarrow (\mathbf{R}/(\mathbf{R}+\mathbf{Q}))$ )

$$R_{12} = R_{11} + R_{21}, \quad R_{22} = \frac{R_{11}^2}{R_{21}} + R_{11}, \quad Q_{22} = \frac{Q_{21} R_{21}^2}{(R_{11} + R_{21})^2}$$

Transformations formulae ( $\mathbf{R}/(\mathbf{R}+\mathbf{Q}) \rightarrow (\mathbf{R}+(\mathbf{R}/\mathbf{Q}))$ )

$$Q_{21} = \frac{Q_{22} (R_{12} + R_{22})^2}{R_{12}^2}, \quad R_{11} = \frac{R_{12} R_{22}}{R_{12} + R_{22}}, \quad R_{21} = \frac{R_{12}^2}{R_{12} + R_{22}}$$

1.7. TRANSFORMATION FORMULAE BETWEEN  $(R+(R/Q))$  AND  $(R/(R+Q))$  11

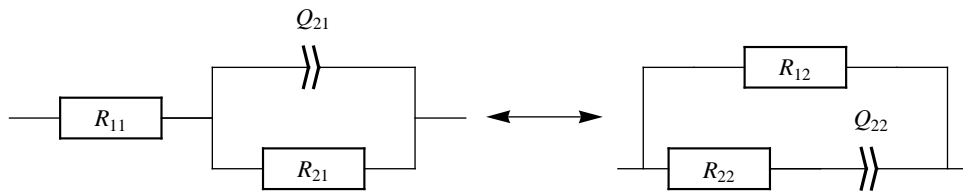


Figure 1.11: The  $(R+(R/Q))$  and  $(R/(R+Q))$  circuits are non-distinguishable for  $\alpha_{21} = \alpha_{22}$  [1].

**1.7.2**  $\alpha_{21} \neq \alpha_{22}$

The  $(R+(R/Q))$  and  $(R/(R+Q))$  circuits (Fig. 1.11) are distinguishable for  $\alpha_{21} \neq \alpha_{22}$



## Chapter 2

# Circuits made of two CPEs

### 2.1 Circuit ( $Q_1+Q_2$ )

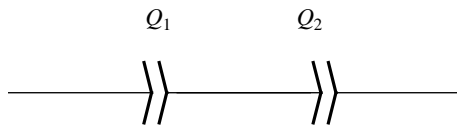


Figure 2.1: Circuit ( $Q_1+Q_2$ ).

#### 2.1.1 $\alpha_1 = \alpha_2 = \alpha$

$$Z(\omega) = \left( \frac{1}{Q_1} + \frac{1}{Q_2} \right) \frac{1}{(i\omega)^\alpha} = \frac{1}{Q} \frac{1}{(i\omega)^\alpha}, \quad Q = \frac{Q_1 Q_2}{Q_1 + Q_2}$$

cf. § 1.1.

#### 2.1.2 $\alpha_1 \neq \alpha_2$

**Impedance**

$$Z(\omega) = \frac{1}{Q_1 (i\omega)^{\alpha_1}} + \frac{1}{Q_2 (i\omega)^{\alpha_2}} = \frac{Q_1 (i\omega)^{\alpha_1} + Q_2 (i\omega)^{\alpha_2}}{Q_1 Q_2 (i\omega)^{\alpha_1 + \alpha_2}}$$

$$\operatorname{Re} Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_1}{2}\right) \omega^{-\alpha_1}}{Q_1} + \frac{\cos\left(\frac{\pi\alpha_2}{2}\right) \omega^{-\alpha_2}}{Q_2}$$

$$\operatorname{Im} Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right) \omega^{-\alpha_1}}{Q_1} - \frac{\sin\left(\frac{\pi\alpha_2}{2}\right) \omega^{-\alpha_2}}{Q_2}$$

$$|Z_{Q_1}| = |Z_{Q_2}| \Rightarrow \omega = \omega_c = \left( \frac{Q_2}{Q_1} \right)^{\frac{1}{\alpha_1 - \alpha_2}}$$

- $\alpha_1 < \alpha_2$  (Figs. 2.2 and 2.3)

$$\omega \rightarrow 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_2 (i\omega)^{\alpha_2}}, \quad \omega \rightarrow \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_1 (i\omega)^{\alpha_1}}$$

- $\alpha_1 > \alpha_2$

$$\omega \rightarrow 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_1 (i\omega)^{\alpha_1}}, \quad \omega \rightarrow \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_2 (i\omega)^{\alpha_2}}$$

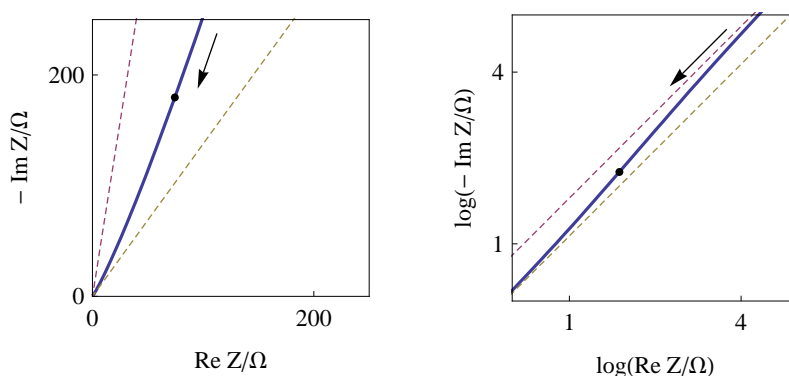


Figure 2.2: Nyquist and log Nyquist [4] diagrams of the impedance for the  $(Q_1+Q_2)$  circuit, plotted for  $Q_1 = 10^{-2} \text{ F cm}^{-2} \text{ s}^{\alpha_1-1}$ ,  $Q_2 = 10^{-2} \text{ F cm}^{-2} \text{ s}^{\alpha_2-1}$ ,  $\alpha_1 = 0.6$ ,  $\alpha_2 = 0.9$  ( $\alpha_1 < \alpha_2$ ). Dots:  $\omega_c = (Q_2/Q_1)^{1/(\alpha_1-\alpha_2)}$ .

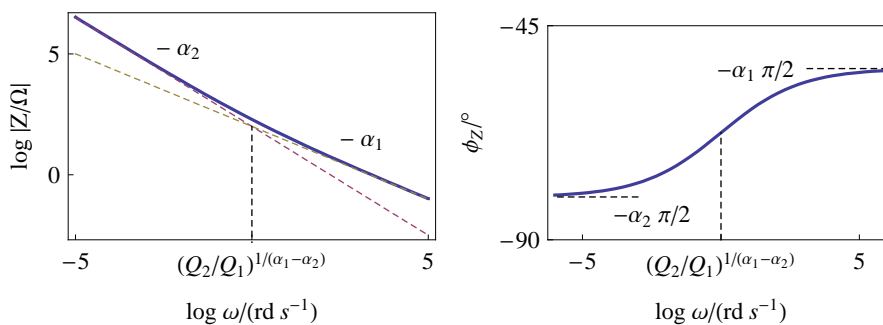


Figure 2.3: Bode diagrams of the impedance for the  $(Q_1+Q_2)$  circuit. Same values of parameters as in Fig. 2.2.  $\alpha_1 < \alpha_2$ .

### 2.1.3 Reduced impedance

$$Z^*(u) = Q_1 \omega_c^{\alpha_1} Z(\omega) = \frac{1}{(iu)^{\alpha_1}} + \frac{1}{(iu)^{\alpha_2}}, \quad u = \frac{\omega}{\omega_c}$$

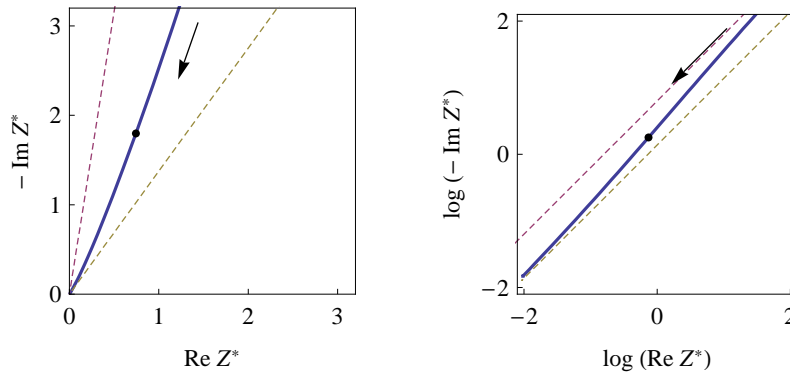


Figure 2.4: Nyquist and log Nyquist [4] diagrams of the reduced impedance for the ( $Q_1+Q_2$ ) circuit, plotted for  $\alpha_1 = 0.6, \alpha_2 = 0.9$  ( $\alpha_1 < \alpha_2$ ). Dots:  $u_c = 1$ .

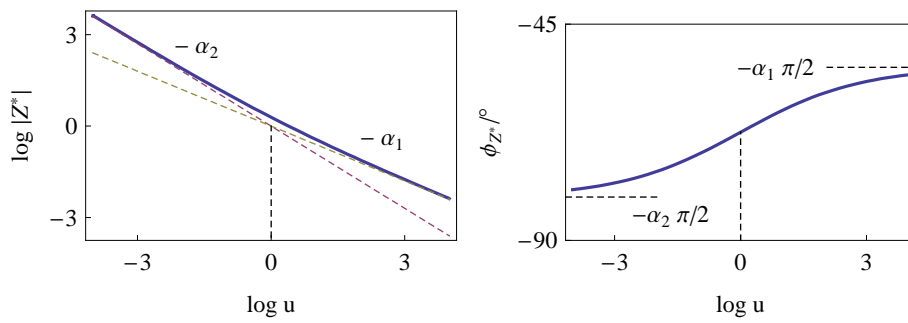
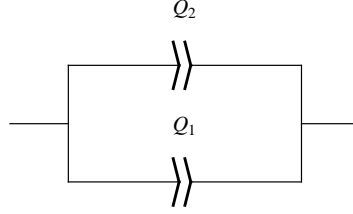


Figure 2.5: Bode diagrams of the impedance for the ( $Q_1+Q_2$ ) circuit. Same values of parameters as in Fig. 2.4.  $\alpha_1 < \alpha_2$ .

## 2.2 Circuit ( $Q_1/Q_2$ )

Figure 2.6: Circuit ( $Q_1/Q_2$ ).

### 2.2.1 $\alpha_1 = \alpha_2 = \alpha$

$$Z(\omega) = \frac{1}{(Q_1 + Q_2)(i\omega)^\alpha} = \frac{1}{Q(i\omega)^\alpha}, \quad Q = Q_1 + Q_2$$

cf. § 1.1.

### 2.2.2 $\alpha_1 \neq \alpha_2$

#### Impedance

$$Z(\omega) = \frac{1}{Q_1(i\omega)^{\alpha_1} + Q_2(i\omega)^{\alpha_2}}$$

$$\operatorname{Re} Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_1}{2}\right) Q_1 \omega^{\alpha_1} + \cos\left(\frac{\pi\alpha_2}{2}\right) Q_2 \omega^{\alpha_2}}{Q_1^2 \omega^{2\alpha_1} + Q_2^2 \omega^{2\alpha_2} + 2 \cos\left(\frac{1}{2}\pi(\alpha_1 - \alpha_2)\right) Q_1 Q_2 \omega^{\alpha_1 + \alpha_2}}$$

$$\operatorname{Im} Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right) Q_1 \omega^{\alpha_1} + \sin\left(\frac{\pi\alpha_2}{2}\right) Q_2 \omega^{\alpha_2}}{Q_1^2 \omega^{2\alpha_1} + Q_2^2 \omega^{2\alpha_2} + 2 \cos\left(\frac{1}{2}\pi(\alpha_1 - \alpha_2)\right) Q_1 Q_2 \omega^{\alpha_1 + \alpha_2}}$$

- $\alpha_1 < \alpha_2$  (Figs. 2.7 and 2.8)

$$\omega \rightarrow 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_1(i\omega)^{\alpha_1}}, \quad \omega \rightarrow \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_2(i\omega)^{\alpha_2}}$$

- $\alpha_1 > \alpha_2$

$$\omega \rightarrow 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_2(i\omega)^{\alpha_2}}, \quad \omega \rightarrow \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_1(i\omega)^{\alpha_1}}$$

### 2.2.3 Reduced impedance

$$Z^*(u) = Q_1 \omega_c^{\alpha_1} Z(\omega) = \frac{1}{(iu)^{\alpha_1} + (iu)^{\alpha_2}}, \quad u = \frac{\omega}{\omega_c}$$

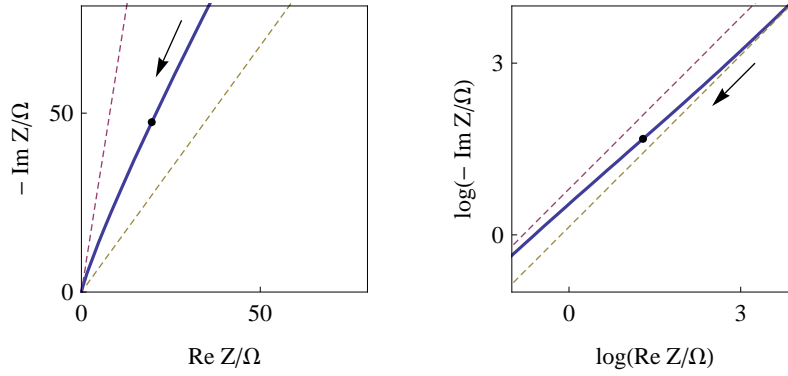


Figure 2.7: Nyquist and log Nyquist [4] diagrams of the impedance for the ( $Q_1/Q_2$ ) circuit plotted for  $Q_1 = 10^{-2} \text{ F cm}^{-2} \text{ s}^{\alpha_1-1}$ ,  $Q_2 = 10^{-2} \text{ F cm}^{-2} \text{ s}^{\alpha_2-1}$ ,  $\alpha_1 = 0.6$ ,  $\alpha_2 = 0.9$  ( $\alpha_1 < \alpha_2$ ). Dots:  $\omega_c = (Q_2/Q_1)^{1/(\alpha_1-\alpha_2)}$ .

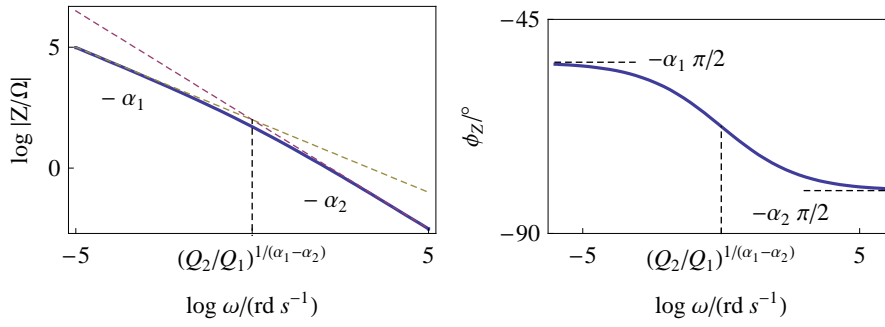


Figure 2.8: Bode diagrams of the impedance for the ( $Q_1/Q_2$ ) circuit. Same values of parameters as in Fig. 2.7.  $\alpha_1 < \alpha_2$ .

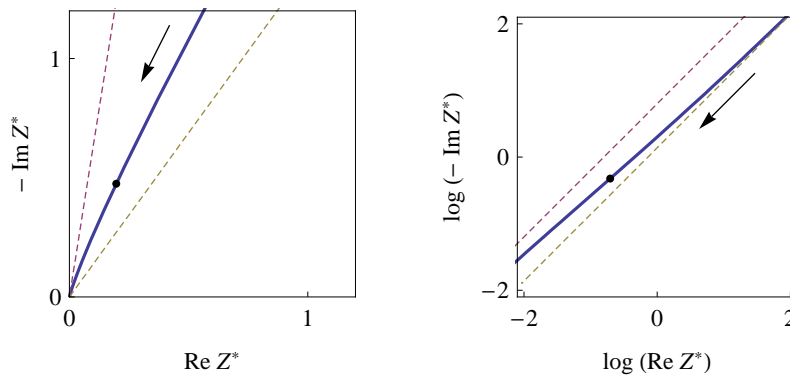


Figure 2.9: Nyquist and log Nyquist [4] diagrams of the reduced impedance for the ( $Q_1/Q_2$ ) circuit, plotted for  $\alpha_1 = 0.6$ ,  $\alpha_2 = 0.9$  ( $\alpha_1 < \alpha_2$ ). Dots:  $u_c = 1$ .

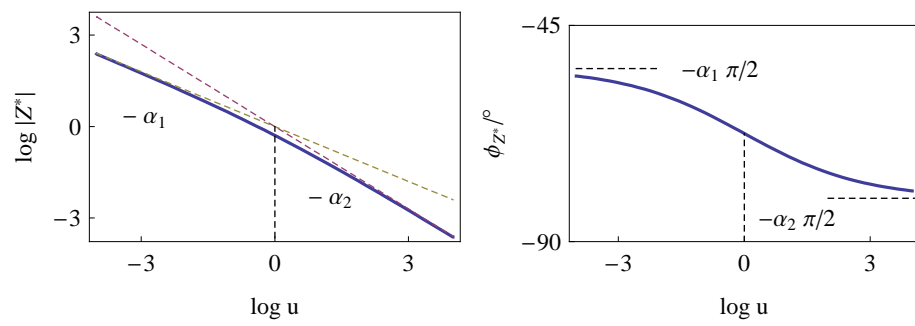


Figure 2.10: Bode diagrams of the impedance for the  $(Q_1/Q_2)$  circuit. Same values of parameters as in Fig. 2.9.  $\alpha_1 < \alpha_2$ .

## Chapter 3

# Circuits made of one R and two CPEs

### 3.1 Circuit $((R_1/Q_1) + Q_2)$

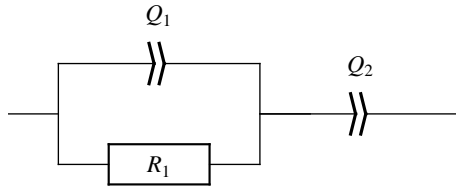


Figure 3.1: Circuit  $((R_1/Q_1)+Q_2)$ .

#### 3.1.1 $\alpha_1 = \alpha_2 = \alpha$

##### Impedance

$$Z(\omega) = \frac{1}{\frac{1}{R_1} + Q_1 (i\omega)^\alpha} + \frac{1}{Q_2 (i\omega)^\alpha}$$

$$Z(\omega) = \frac{1 + (i\omega)^\alpha \tau_2}{(i\omega)^\alpha Q_2 (1 + (i\omega)^\alpha \tau_1)}, \quad \tau_1 = R_1 Q_1, \quad \tau_2 = (Q_1 + Q_2) R_1, \quad \tau_1 < \tau_2$$

$$\operatorname{Re} Z(\omega) = -\frac{\cos\left(\frac{\pi\alpha}{2}\right) (\tau_1 \tau_2 \omega^{2\alpha} + 1) \omega^{-\alpha} + \cos(\pi\alpha) \tau_1 + \tau_2}{Q_2 (\tau_1 (\tau_1 \omega^\alpha + 2 \cos\left(\frac{\pi\alpha}{2}\right)) \omega^\alpha + 1)}$$

$$\operatorname{Im} Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha}{2}\right) (\tau_1 \tau_2 \omega^{2\alpha} + 1) \omega^{-\alpha} + \sin(\pi\alpha) \tau_1}{Q_2 (\tau_1 (\tau_1 \omega^\alpha + 2 \cos\left(\frac{\pi\alpha}{2}\right)) \omega^\alpha + 1)}$$

**Reduced impedance**

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{1}{T-1} \frac{1 + T(iu)^\alpha}{(iu)^\alpha (1 + (iu)^\alpha)} \quad (3.1)$$

$$u = \omega \tau^{1/\alpha}, \quad T = \tau_2/\tau_1 = 1 + Q_2/Q_1 > 1$$

$$\operatorname{Re} Z^*(u) = \frac{u^{-\alpha} \left( (T + \cos(\alpha\pi))u^\alpha + (Tu^{2\alpha} + 1) \cos\left(\frac{\alpha\pi}{2}\right) \right)}{(T-1) \left( 2 \cos\left(\frac{\alpha\pi}{2}\right) u^\alpha + u^{2\alpha} + 1 \right)}$$

$$\operatorname{Im} Z^*(u) = u^{-\alpha} \left( \frac{1}{1-T} - \frac{u^{2\alpha}}{2 \cos\left(\frac{\alpha\pi}{2}\right) u^\alpha + u^{2\alpha} + 1} \right) \sin\left(\frac{\alpha\pi}{2}\right)$$

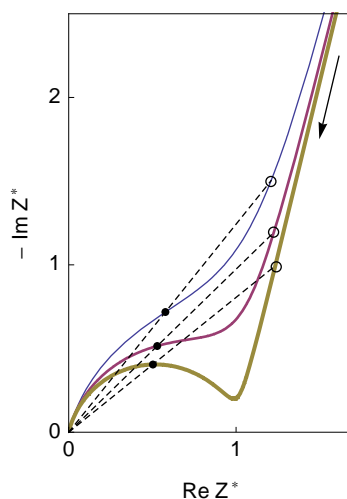


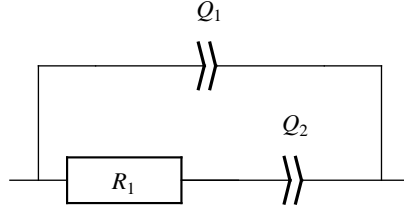
Figure 3.2: Nyquist diagram of the reduced impedance for the  $((R_1/Q_1)+Q_2)$  circuit (Fig. 3.1, Eq. (3.1)), plotted for  $T = 4, 9, 90$  and  $\alpha = 0.85$ . The line thickness increases with increasing  $T$ . Dots: reduced characteristic angular frequency  $u_{c1} = 1$ ; circles: reduced characteristic angular frequency  $u_{c2} = 1/T^{1/\alpha}$  ( $\phi_{u_{c1}} = \phi_{u_{c2}}$ ).

**3.1.2  $\alpha_1 \neq \alpha_2$** **Impedance**

$$Z(\omega) = \frac{1}{\frac{1}{R_1} + Q_1 (i\omega)^{\alpha_1}} + \frac{1}{Q_2 (i\omega)^{\alpha_2}}$$

$$\operatorname{Re} Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_2}{2}\right) \omega^{-\alpha_2}}{Q_2} + \frac{R_1 \left( \cos\left(\frac{\pi\alpha_1}{2}\right) Q_1 R_1 \omega^{\alpha_1} + 1 \right)}{Q_1 R_1 \left( Q_1 R_1 \omega^{\alpha_1} + 2 \cos\left(\frac{\pi\alpha_1}{2}\right) \right) \omega^{\alpha_1} + 1}$$

$$\operatorname{Im} Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right) Q_1 R_1^2 \omega^{\alpha_1}}{Q_1 R_1 \left( Q_1 R_1 \omega^{\alpha_1} + 2 \cos\left(\frac{\pi\alpha_1}{2}\right) \right) \omega^{\alpha_1} + 1} - \frac{\sin\left(\frac{\pi\alpha_2}{2}\right) \omega^{-\alpha_2}}{Q_2}$$

Figure 3.3: Circuit  $((R_1+Q_2)/Q_1)$ .

### 3.2 Circuit $((R_1 + Q_1)/Q_2)$

#### 3.2.1 $\alpha_1 = \alpha_2 = \alpha$

##### Impedance

$$Z(\omega) = \frac{1}{(i\omega)^\alpha Q_1 + \frac{1}{R_1 + \frac{1}{(i\omega)^\alpha Q_2}}} = \frac{1 + Q_2 R_1 (i\omega)^\alpha}{(i\omega)^\alpha (Q_1 + Q_2) \left(1 + \frac{(i\omega)^\alpha Q_1 Q_2 R_1}{Q_1 + Q_2}\right)}$$

$$Z(\omega) = \frac{1 + \tau_2 (i\omega)^\alpha}{(i\omega)^\alpha (Q_1 + Q_2) (1 + (i\omega)^\alpha \tau_1)}, \quad \tau_1 = \frac{Q_1 Q_2 R_1}{Q_1 + Q_2}, \quad \tau_2 = Q_2 R_1$$

$$\operatorname{Re} Z(\omega) = \frac{\omega^{-\alpha} (\cos(\pi\alpha)\omega^\alpha + \tau_2\omega^\alpha + \cos(\frac{\pi\alpha}{2}) (\tau_2\omega^{2\alpha} + 1))}{(2 \cos(\frac{\pi\alpha}{2}) \omega^\alpha + \omega^{2\alpha} + 1) (Q_1 + Q_2) \tau_1}$$

$$\operatorname{Im} Z(\omega) = -\frac{\omega^{-\alpha} \sin(\frac{\pi\alpha}{2}) (2 \cos(\frac{\pi\alpha}{2}) \omega^\alpha + \tau_2\omega^{2\alpha} + 1)}{(2 \cos(\frac{\pi\alpha}{2}) \omega^\alpha + \omega^{2\alpha} + 1) (Q_1 + Q_2) \tau_1}$$

##### Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{T-1}{T^2} \frac{1 + T (iu)^\alpha}{(iu)^\alpha (1 + (iu)^\alpha)} \quad (3.2)$$

$$u = \omega \tau^{1/\alpha}, \quad T = \tau_2/\tau_1 = 1 + Q_2/Q_1 > 1$$

$$\operatorname{Re} Z^*(u) = \frac{(T-1)u^{-\alpha} ((T + \cos(\alpha\pi))u^\alpha + (Tu^{2\alpha} + 1) \cos(\frac{\alpha\pi}{2}))}{T^2 (2 \cos(\frac{\alpha\pi}{2}) u^\alpha + u^{2\alpha} + 1)}$$

$$\operatorname{Im} Z^*(u) = -\frac{(T-1)u^{-\alpha} (2 \cos(\frac{\alpha\pi}{2}) u^\alpha + Tu^{2\alpha} + 1) \sin(\frac{\alpha\pi}{2})}{T^2 (2 \cos(\frac{\alpha\pi}{2}) u^\alpha + u^{2\alpha} + 1)}$$

#### 3.2.2 $\alpha_1 \neq \alpha_2$

$$Z(\omega) = \frac{\frac{1}{(i\omega)^{\alpha_2} Q_2} + R_1}{(i\omega)^{\alpha_1} Q_1 \left(\frac{1}{(i\omega)^{\alpha_1} Q_1} + \frac{1}{(i\omega)^{\alpha_2} Q_2} + R_1\right)}$$

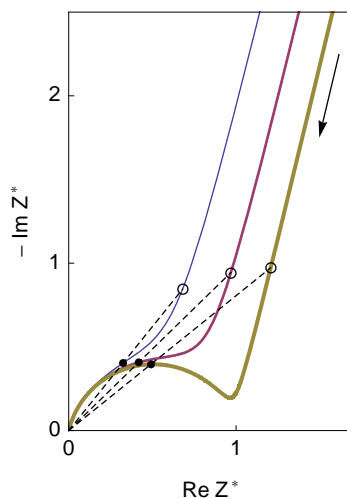


Figure 3.4: Nyquist diagram of the reduced impedance for the  $((R_1+Q_1)/Q_2)$  circuit (Fig. 3.3, Eq. (3.2)), plotted for  $T = 4, 9, 90$  and  $\alpha = 0.85$ . The line thickness increases with increasing  $T$ . Dots: reduced characteristic angular frequency  $u_{c1} = 1$ ; circles: reduced characteristic angular frequency  $u_{c2} = 1/T^{1/\alpha}$  ( $\phi_{u_{c1}} = \phi_{u_{c2}}$ ).

$$Z(\omega) = \frac{1 + \tau (i\omega)^{\alpha_2}}{(i\omega)^{\alpha_1} Q_1 + (i\omega)^{\alpha_2} Q_2 + \tau (i\omega)^{\alpha_1 + \alpha_2} Q_1}, \quad \tau = R_1 Q_2$$

$$\begin{aligned} \text{Re } Z(\omega) = & \\ & \frac{(\omega^{\alpha_1} c_{\alpha_1} (1 + \tau^2 \omega^{2\alpha_2} + 2\tau \omega^{\alpha_2} c_{\alpha_2}) Q_1 + \omega^{\alpha_2} (\tau \omega^{\alpha_2} + c_{\alpha_2}) Q_2)}{(\omega^{2\alpha_1} (1 + \tau^2 \omega^{2\alpha_2} + 2\tau \omega^{\alpha_2} c_{\alpha_2}) Q_1^2 + 2\omega^{\alpha_1 + \alpha_2} (\tau \omega^{\alpha_2} c_{\alpha_1} + c_{\alpha_1 m \alpha_2}) Q_1 Q_2 + \omega^{2\alpha_2} Q_2^2)} \end{aligned}$$

$$c_{\alpha_1 m \alpha_2} = \cos\left(\frac{\pi (\alpha_1 - \alpha_2)}{2}\right)$$

$$\begin{aligned} \text{Im } Z(\omega) = & \\ & \frac{(-\omega^{\alpha_1} (1 + \tau^2 \omega^{2\alpha_2} + 2\tau \omega^{\alpha_2} c_{\alpha_2}) Q_1 s_{\alpha_1} - \omega^{\alpha_2} Q_2 s_{\alpha_2})}{(\omega^{2\alpha_1} (1 + \tau^2 \omega^{2\alpha_2} + 2\tau \omega^{\alpha_2} c_{\alpha_2}) Q_1^2 + 2\omega^{\alpha_1 + \alpha_2} (\tau \omega^{\alpha_2} c_{\alpha_1} + c_{\alpha_1 m \alpha_2}) Q_1 Q_2 + \omega^{2\alpha_2} Q_2^2)} \end{aligned}$$

## Chapter 4

# Circuits made of two Rs and two CPEs

### 4.1 Circuit $((R_1/Q_1)+(R_2/Q_2))$

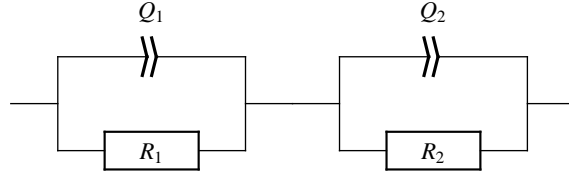


Figure 4.1: Circuit  $((R_1/Q_1)+(R_2/Q_2))$ .

$$Z(\omega) = \frac{1}{(i\omega)^{\alpha_1} Q_1 + \frac{1}{R_1}} + \frac{1}{(i\omega)^{\alpha_2} Q_2 + \frac{1}{R_2}}$$

$$Z(\omega) = \frac{R_1}{1 + (i\omega)^{\alpha_1} \tau_1} + \frac{R_2}{1 + (i\omega)^{\alpha_2} \tau_2}, \quad \tau_1 = R_1 Q_1, \quad \tau_2 = R_2 Q_2$$

$$Z(\omega) = \frac{R_1 + R_2 + (i\omega)^{\alpha_1} R_2 \tau_1 + (i\omega)^{\alpha_2} R_1 \tau_2}{(1 + (i\omega)^{\alpha_1} \tau_1) (1 + (i\omega)^{\alpha_2} \tau_2)}$$

$$\text{Re } Z(\omega) = \frac{R_1 (1 + \omega^{\alpha_1} c_{\alpha_1} \tau_1)}{1 + \omega^{\alpha_1} \tau_1 (2 c_{\alpha_1} + \omega^{\alpha_1} \tau_1)} + \frac{R_2 (1 + \omega^{\alpha_2} c_{\alpha_2} \tau_2)}{1 + \omega^{\alpha_2} \tau_2 (2 c_{\alpha_2} + \omega^{\alpha_2} \tau_2)}$$

$$\text{Im } Z(\omega) = -\frac{\omega^{\alpha_1} R_1 s_{\alpha_1} \tau_1}{1 + \omega^{\alpha_1} \tau_1 (2 c_{\alpha_1} + \omega^{\alpha_1} \tau_1)} - \frac{\omega^{\alpha_2} R_2 s_{\alpha_2} \tau_2}{1 + \omega^{\alpha_2} \tau_2 (2 c_{\alpha_2} + \omega^{\alpha_2} \tau_2)}$$

## 4.2 Circuit $((R_1 + (R_2/Q_2))/Q_1)$

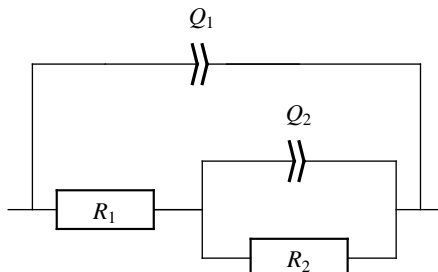


Figure 4.2: Circuit  $((R_1 + (R_2/Q_2))/Q_1)$ .

$$Z(\omega) = \frac{1}{(i\omega)^{\alpha_1} Q_1 + \frac{1}{R_1 + \frac{1}{(i\omega)^{\alpha_2} Q_2 + \frac{1}{R_2}}}}$$

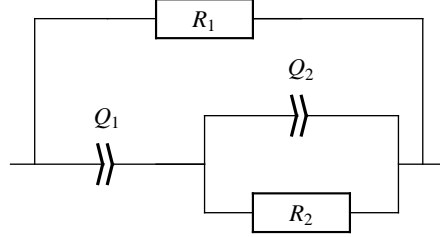
$$Z(\omega) = \frac{R_1 + R_2 + (i\omega)^{\alpha_2} Q_2 R_1 R_2}{1 + (i\omega)^{\alpha_1} Q_1 (R_1 + R_2) + (i\omega)^{\alpha_2} Q_2 R_2 + (i\omega)^{\alpha_1 + \alpha_2} Q_1 Q_2 R_1 R_2}$$

$$\begin{aligned} \text{Re } Z(\omega) = & (R_1 + R_2 + \omega^{2\alpha_2} Q_2^2 R_1 (1 + \omega^{\alpha_1} C_{\alpha_1} Q_1 R_1) R_2^2 + \\ & \omega^{\alpha_1} C_{\alpha_1} Q_1 (R_1 + R_2)^2 + \omega^{\alpha_2} C_{\alpha_2} Q_2 R_2 (R_2 + 2 R_1 (1 + \omega^{\alpha_1} C_{\alpha_1} Q_1 (R_1 + R_2)))) / \\ & (1 + \omega^{2\alpha_2} Q_2^2 (1 + \omega^{\alpha_1} Q_1 R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 R_1)) R_2^2 + \\ & \omega^{\alpha_1} Q_1 (R_1 + R_2) (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)) + 2 \omega^{\alpha_2} Q_2 R_2 \\ & \times (C_{\alpha_2} + \omega^{\alpha_1} Q_1 (C_{\alpha_1 m \alpha_2} R_2 + C_{\alpha_2} R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)))))) \end{aligned}$$

$$c_{\alpha_1 m \alpha_2} = \cos\left(\frac{\pi (\alpha_1 - \alpha_2)}{2}\right)$$

$$\begin{aligned} \text{Im } Z(\omega) = & (\omega^{\alpha_1} Q_1 (-\omega^{2\alpha_2} Q_2^2 R_1^2 R_2^2 - 2 \omega^{\alpha_2} C_{\alpha_2} Q_2 R_1 R_2 (R_1 + R_2) - \\ & (R_1 + R_2)^2) S_{\alpha_1} - \omega^{\alpha_2} Q_2 R_2^2 S_{\alpha_2}) / \\ & (1 + \omega^{2\alpha_2} Q_2^2 (1 + \omega^{\alpha_1} Q_1 R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 R_1)) R_2^2 + \\ & \omega^{\alpha_1} Q_1 (R_1 + R_2) (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)) + 2 \omega^{\alpha_2} Q_2 R_2 \\ & \times (C_{\alpha_2} + \omega^{\alpha_1} Q_1 (C_{\alpha_1 m \alpha_2} R_2 + C_{\alpha_2} R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)))))) \end{aligned}$$

### 4.3 Circuit $((Q_1+(R_2/Q_2))/R_1)$

Figure 4.3: Circuit  $((Q_1+(R_2/Q_2))/R_1)$ .

$$Z(\omega) = \frac{1}{\frac{1}{R_1} + \frac{1}{\frac{1}{(i\omega)^{\alpha_1} Q_1} + \frac{1}{(i\omega)^{\alpha_2} Q_2 + \frac{1}{R_2}}}}$$

$$Z(\omega) = \frac{R_1 (1 + (i\omega)^{\alpha_1} Q_1 R_2 + (i\omega)^{\alpha_2} Q_2 R_2)}{1 + (i\omega)^{\alpha_1} Q_1 (R_1 + R_2) + (i\omega)^{\alpha_2} Q_2 R_2 + (i\omega)^{\alpha_1 + \alpha_2} Q_1 Q_2 R_1 R_2}$$

$$\begin{aligned} \text{Re } Z(\omega) = & (R_1 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2) + \omega^{2\alpha_1} Q_1^2 R_2 \\ & \times (R_2 + R_1 (1 + \omega^{\alpha_2} C_{\alpha_2} Q_2 R_2)) + \omega^{\alpha_1} Q_1 (2 R_2 (C_{\alpha_1} + \omega^{\alpha_2} C_{\alpha_1 m \alpha_2} Q_2 R_2) + \\ & C_{\alpha_1} R_1 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2)))) / \\ & (1 + \omega^{2\alpha_2} Q_2^2 (1 + \omega^{\alpha_1} Q_1 R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 R_1)) R_2^2 + \\ & \omega^{\alpha_1} Q_1 (R_1 + R_2) (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)) + \\ & 2 \omega^{\alpha_2} Q_2 R_2 (C_{\alpha_2} + \omega^{\alpha_1} Q_1 (C_{\alpha_1 m \alpha_2} R_2 + C_{\alpha_2} R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)))))) \end{aligned}$$

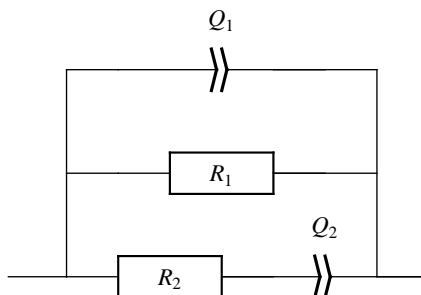
$$\begin{aligned} \text{Im } Z(\omega) = & -\omega^{\alpha_1} Q_1 R_2^2 (S_{\alpha_1} + \omega^{\alpha_2} Q_2 R_2 ((2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2) S_{\alpha_1} + \omega^{\alpha_1} Q_1 R_2 S_{\alpha_2})) / \\ & (1 + \omega^{2\alpha_2} Q_2^2 (1 + \omega^{\alpha_1} Q_1 R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 R_1)) R_2^2 + \\ & \omega^{\alpha_1} Q_1 (R_1 + R_2) (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)) + \\ & 2 \omega^{\alpha_2} Q_2 R_2 (C_{\alpha_2} + \omega^{\alpha_1} Q_1 (C_{\alpha_1 m \alpha_2} R_2 + C_{\alpha_2} R_1 (2 C_{\alpha_1} + \omega^{\alpha_1} Q_1 (R_1 + R_2)))))) \end{aligned}$$

$$Z(\omega) = \frac{R_1 (1 + \tau_1 (i\omega)^{\alpha_1} + \tau_2 (i\omega)^{\alpha_2})}{1 + (1 + R_1/R_2) \tau_1 (i\omega)^{\alpha_1} + \tau_2 (i\omega)^{\alpha_2} + \tau_1 \tau_2 (R_1/R_2) (i\omega)^{\alpha_1 + \alpha_2}}$$

$$\tau_1 = Q_1 R_2, \tau_2 = Q_2 R_2$$

### 4.4 Circuit $((Q_2+R_2)/R_1)/Q_1)$

$$Z(\omega) = \frac{1}{(i\omega)^{\alpha_1} Q_1 + \frac{1}{R_1} + \frac{1}{\frac{1}{(i\omega)^{\alpha_2} Q_2} + R_2}}$$

Figure 4.4: Circuit  $((Q_2+R_2)/R_1)/Q_1$ .

$$Z(\omega) = \frac{R_1 (1 + (i\omega)^{\alpha_2} Q_2 R_2)}{1 + (i\omega)^{\alpha_1} Q_1 R_1 + (i\omega)^{\alpha_2} Q_2 R_1 + (i\omega)^{\alpha_2} Q_2 R_2 + (i\omega)^{\alpha_1 + \alpha_2} Q_1 Q_2 R_1 R_2}$$

$$\begin{aligned} \text{Re } Z(\omega) = & (R_1 (1 + \omega^{\alpha_2} Q_2 (\omega^{\alpha_2} Q_2 R_2 (R_1 + R_2) + C_{\alpha_2} (R_1 + 2 R_2)) + \\ & \omega^{\alpha_1} C_{\alpha_1} Q_1 R_1 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2)))) / \\ & (1 + \omega^{\alpha_2} Q_2 (R_1 + R_2) (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 (R_1 + R_2)) + \\ & \omega^{2\alpha_1} Q_1^2 R_1^2 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2)) + 2\omega^{\alpha_1} Q_1 R_1 \\ & \times (C_{\alpha_1} + \omega^{\alpha_2} Q_2 (C_{\alpha_1 m \alpha_2} R_1 + 2 C_{\alpha_1} C_{\alpha_2} R_2 + \omega^{\alpha_2} C_{\alpha_1} Q_2 R_2 (R_1 + R_2)))) \end{aligned}$$

$$\begin{aligned} \text{Im } Z(\omega) = & (R_1^2 (- (\omega^{\alpha_1} Q_1 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2)) S_{\alpha_1}) - \omega^{\alpha_2} Q_2 S_{\alpha_2})) / \\ & (1 + \omega^{\alpha_2} Q_2 (R_1 + R_2) (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 (R_1 + R_2)) + \\ & \omega^{2\alpha_1} Q_1^2 R_1^2 (1 + \omega^{\alpha_2} Q_2 R_2 (2 C_{\alpha_2} + \omega^{\alpha_2} Q_2 R_2)) + 2\omega^{\alpha_1} Q_1 R_1 \\ & \times (C_{\alpha_1} + \omega^{\alpha_2} Q_2 (C_{\alpha_1 m \alpha_2} R_1 + 2 C_{\alpha_1} C_{\alpha_2} R_2 + \omega^{\alpha_2} C_{\alpha_1} Q_2 R_2 (R_1 + R_2)))) \end{aligned}$$

$$Z(\omega) = \frac{R_1 (1 + (i\omega)^{\alpha_2} \tau_2)}{1 + (i\omega)^{\alpha_1} \tau_1 + (1 + R_1/R_2) (i\omega)^{\alpha_2} \tau_2 + (i\omega)^{\alpha_1 + \alpha_2} \tau_1 \tau_2}$$

$$\tau_1 = Q_1 R_1, \tau_2 = Q_2 R_2$$

# Appendix A

## Symbols for CPE

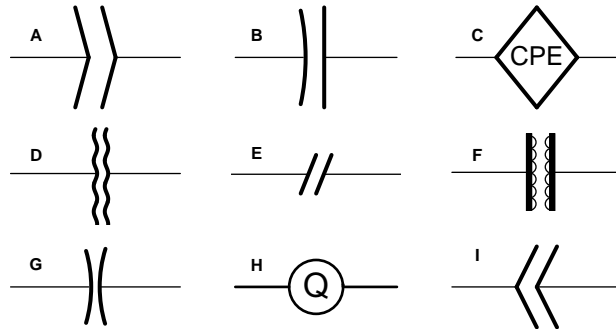


Figure A.1: Some CPE symbols, taken from A: [7], B: [10], C: [13], D: [2], E: [5], F: [8], G: [9], H: [11], I: [6].



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